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Large N WZW Field Theory Of $N = 2$ Strings ¹

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Abstract

We explore the quantum properties of self-dual gravity formulated as a large N two-dimensional WZW sigma model. Using a non-trivial classical background, we show that a $(2, 2)$ space-time is generated. The theory contains an infinite series of higher point vertices. At tree level we show that, in spite of the presence of higher than cubic vertices, the on-shell 4 and higher point functions vanish, indicating that this model is related with the field theory of closed $N = 2$ strings. We examine the one-loop on-shell 3-point amplitude and show that it is ultra-violet finite.

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1 Introduction

String theory through its worldsheet description exhibits notable finiteness properties. Its target space description in terms of field theory is then of major interest both with regard to nonperturbative studies, but more recently also as a building block of higher dimensional extended objects. One of the simplest models is given by the $N = 2$ string [1]–[9]. This theory, due to its solvability, has been extensively studied since its discovery [1, 2] and served as a laboratory for acquiring deeper insight into the structure of string theory.

The main features of the $N = 2$ string are the appearance of a four-dimensional $(2, 2)$ target space and the vanishing of all higher point amplitudes [4]–[9]. Its field theoretic description seems to be given by versions of self-dual Yang-Mills theory and self-dual gravity. These field theoretic representations are subjects of major interest and are at present known to contain a number of puzzles. In particular, at the quantum level there is as yet no clear way of defining self-dual gravity, the theory is accompanied by infinities and it is in fact string theory that may suggest the proper definition of these four-dimensional theories. The interest in these theories has increased recently [10, 11] since through the worldsheet from target space mechanism [12, 11] they are promising candidates for building blocks for a worldsheet description of membranes. Indeed, a field theoretic formulation [10] of the membrane matrix model was seen to be given by an $SU(\infty)$ WZW theory in two dimensions. Since the additional worldsheet coordinates appear from the large N color degrees of freedom [13, 14] one finds a similarity with various sigma model descriptions of self-dual gravity [15]–[19]. This approach to self-dual gravity has so far been explored strictly at the classical level: the equations of motion when written in terms of currents lead to Plebański’s heavenly equations [15], with different sigma models being classically equivalent. One might envision a direct quantization of the cubic Plebański action, but some studies of non-abelian duality transformations for sigma models [20] would indicate that this could

exhibit problems, such as the wrong beta function.

In this paper, we approach the problem of quantizing self-dual gravity and developing a field theory of $N = 2$ closed strings based directly on a (perturbed) WZW model represented as a non-linear field theory with an infinite series of higher point vertices. The fact that the WZW model possesses remarkable quantum properties (such as vanishing beta function) offers the hope that the corresponding gravitational theory will be equally well defined. We expect that the infinite sequence of higher point vertices is likely to be relevant for its finiteness. The first question at hand is then to verify the vanishing of higher than 3-point on-shell amplitudes which is what we do in this paper. We also proceed to study the theory at one-loop and demonstrate the vanishing of ultra-violet divergences in the computation of the on-shell 3-point amplitude. We establish a correspondence of our model with a particular large N limit of four-dimensional WZW theory [21, 22].

2 The Model

We will consider a two dimensional sigma model defined in a space-time with Lorentzian signature $(1, 1)$, light-cone coordinates $x^+ = x^1 + x^2$, $x^- = x^1 - x^2$ and metric $ds_{2d}^2 = dx^+ dx^-$. The model will be a chiral model with a Wess-Zumino term, where the coefficient of the chiral term will be perturbed from its conformal fixed point value, so that our model can be seen as a perturbed WZW model at level K ,

$$S(g) = \frac{(1 + \epsilon)K}{4\pi} \int dx^+ dx^- \text{Tr}(\partial_+ g \partial_- g^{-1}) + \frac{K}{12\pi} \Gamma_{WZW}(g), \quad (1)$$

where g is the group valued field, ϵ is a real parameter and $\Gamma(g)$ is the Wess-Zumino term. When $\epsilon = 0$ we obtain the usual WZW model fixed point. The group G remains unspecified for now, but shortly we will take the Lie algebra to be some large N limit of $su(N)$.

The classical equations of motion will be given by

$$\frac{\epsilon}{1+\epsilon} \partial_+(g^{-1}\partial_-g) + \frac{2+\epsilon}{1+\epsilon} \partial_-(g^{-1}\partial_+g) = 0. \quad (2)$$

As long as $\epsilon \neq -2, -1, 0$, one can define Q by

$$\frac{2+\epsilon}{1+\epsilon} g^{-1}\partial_+g = Q^{-1}\partial_+Q \quad \text{and} \quad \frac{\epsilon}{1+\epsilon} g^{-1}\partial_-g = Q^{-1}\partial_-Q$$

where Q is a classical solution of the pure chiral model, see for example [25],

$$\partial_+(Q^{-1}\partial_-Q) + \partial_-(Q^{-1}\partial_+Q) = 0.$$

To make the connection between the two-dimensional sigma model (1) and four-dimensional self-dual gravity, we start by expanding the field g around a specific classical configuration. For reasons that will become clear shortly, this classical background will be

$$Q^{-1}\partial_+Q = \frac{\hat{q}}{\Lambda} + \frac{x^-}{2\Lambda^2}; \quad Q^{-1}\partial_-Q = \frac{\hat{p}}{\Lambda} - \frac{x^+}{2\Lambda^2}, \quad (3)$$

where the matrices \hat{q} and \hat{p} satisfy $[\hat{q}, \hat{p}] = 1$ and Λ is an arbitrary parameter. In the context of our approach, what is important about \hat{q} and \hat{p} is that in the large N limit they become canonical variables q and p , possibly after some rescaling by factors of N . The corresponding g will be given by

$$g_0^{-1}\partial_+g_0 = \frac{1+\epsilon}{2+\epsilon} \left(\frac{\hat{q}}{\Lambda} + \frac{x^-}{2\Lambda^2} \right); \quad g_0^{-1}\partial_-g_0 = \frac{1+\epsilon}{\epsilon} \left(\frac{\hat{p}}{\Lambda} - \frac{x^+}{2\Lambda^2} \right). \quad (4)$$

We will now expand around this classical background. We take $N \rightarrow \infty$ such that the fields will take values in the infinite dimensional Poisson algebra of functions on some two-dimensional space Σ , $sdiff(\Sigma)$. The Lie bracket is then given by the Poisson bracket

$$\{f, g\} = \partial_q f \partial_p g - \partial_p f \partial_q g \quad (5)$$

where f, g are functions of the coordinates q, p on Σ . The trace becomes the integration over the $su(\infty)$ “color” variables q and p . It is well known that this Lie algebra is

closely related to the large N limit(s) of $SU(N)$ [13]. The quantum fluctuations will be described by the field ω , which will take values in the Poisson algebra, where

$$g = g_0 \exp(\omega). \quad (6)$$

From the Polyakov-Wigman formula [23, 24] and after rescaling the field ω to normalize the kinetic term we obtain²

$$\begin{aligned} S(\omega) = & \frac{1}{2} \int dx^+ dx^- \text{Tr}(\partial_p \omega \partial_- \omega - \partial_q \omega \partial_+ \omega) \\ & + \frac{1}{3!} \kappa \int dx^+ dx^- \text{Tr}(\omega \{\partial_p \omega, \partial_- \omega\} - \omega \{\partial_q \omega, \partial_+ \omega\}) \\ & + \frac{1}{4!} \kappa^2 \int dx^+ dx^- \text{Tr}(\partial_q \omega \{\{\partial_+ \omega, \omega\}, \omega\} - \partial_p \omega \{\{\partial_- \omega, \omega\}, \omega\}) + \dots \\ & + \Lambda \int dx^+ dx^- \text{Tr}(-\partial_+ \omega \partial_- \omega - \frac{2}{3!} \kappa \omega \{\partial_+ \omega, \partial_- \omega\} + \frac{2}{4!} \kappa^2 \omega \{\{\partial_- \omega, \omega\}, \partial_+ \omega\}) + \dots \end{aligned} \quad (7)$$

where the coupling constant $\kappa = \sqrt{\frac{4\pi\Lambda}{K(1+\epsilon)}}$. We see that we generate cubic, quartic and higher point vertices for the field ω . Moreover, the two-dimensional propagator $\partial_+ \omega \partial_- \omega$, together with other terms without derivatives in the color directions, is multiplied by factor of Λ . In the limit $\Lambda \rightarrow 0$ these terms disappear and the remaining quadratic terms in ω define a four-dimensional-looking propagator for a metric of $(2, 2)$ signature³

$$ds_{4d}^2 = dx^+ dq - dx^- dp. \quad (8)$$

We will take $\Lambda \rightarrow 0$ for the remainder of the paper, defining a double-scaling limit where the level K and the parameter ϵ are such that κ is finite and nonzero. The cubic vertex of order κ appearing in (7) represents an $SO(2, 2)$ Lorentz transformation of the cubic vertex $\omega \{\partial_+ \omega, \partial_- \omega\}$, whose equation of motion is one of the forms of the

²The x^+ and x^- dependent terms in the classical background currents (4) give vanishing contributions to $S(\omega)$.

³Notice that the fact that we obtain signature $(2, 2)$ is closely related with the choice of sign in the classical background (3).

Plebański equation ⁴ for four-dimensional self-dual gravity. Its amplitudes [17] are closely related to the ones of the closed $N = 2$ string [4]–[9]. However, we note that in the present model there is in addition an infinite series of higher point vertices.

When we compare with the $N = 2$ string field theory of [4] we will identify κ with the closed string coupling constant. We note that at this point we can analytically continue in x^+, x^-, q, p so that we get the flat Kähler metric

$$ds_{Ad}^2 = dyd\bar{y} - dzd\bar{z} \quad (9)$$

where y, \bar{y}, z, \bar{z} are complex coordinates corresponding to x^+, q, x^-, p respectively. The field ω is then related with deformations of the flat Kähler potential [4]. We will use the notation in (9) in the remainder of the paper.

Since in our approach we also generate an infinite series of higher point vertices, in the next section, we will study the amplitudes coming from these vertices and will show that they are also compatible with the $N = 2$ closed string. However, the fact that higher point vertices are present, and the close relation of this theory with the WZW model, leads us to believe that it will have nice properties beyond the one-loop level. Moreover, the action (1) provides a systematic expansion for determining these higher point vertices. It is hoped that the fact that two of the four dimensions appear as large N color variables in disguise, may shed some light into the problem of matching string and field theory amplitudes at one-loop level [4, 5, 6, 9]. In fact, the large N approach suggests a possible infra-red regulator for the momenta along \bar{y}, \bar{z} . There is also the possibility that some specific choice of Σ , for example a torus T^2 , will regulate these momenta even at infinite N .

⁴We note that the Plebański equation is not $SO(2, 2)$ invariant, at least manifestly. For a discussion of the Lorentz symmetries of self-dual gravity see [26].

3 The Amplitudes

In the next two sub-sections we will examine the field theory amplitudes of (1), always with $\Lambda \rightarrow 0$. We will first look at the tree level amplitudes, where we will show that the 4 and 5-point functions vanish on-shell. We will also comment on the relation between (1) and four-dimensional self-dual Yang-Mills theory. We will then look at the one-loop on-shell 3-point function and show that the new 4 and 5-point vertices do not contribute to it.

3.1 Tree Level Amplitudes

The metric is given by (9) and the corresponding components of the momenta k will be $k_y, k_{\bar{y}}, k_z, k_{\bar{z}}$. The inner product will then be given by $k_i \cdot k_j = \overline{k_j} \cdot k_i = k_{iy}k_{j\bar{y}} - k_{iz}k_{j\bar{z}}$.

Following [4, 17] we introduce the kinematical quantities ⁵,

$$\begin{aligned} a_{ij} &= -a_{ji} = k_{iy}k_{jz} - k_{jy}k_{iz} ; & \bar{a}_{ij} &= -\bar{a}_{ji} = k_{i\bar{y}}k_{j\bar{z}} - k_{j\bar{y}}k_{i\bar{z}} ; \\ k_{ij} &= k_i \cdot k_j ; & s_{ij} &= s_{ji} = k_{ij} + k_{ji} ; & c_{ij} &= -c_{ji} = k_{ij} - k_{ji}. \end{aligned} \quad (10)$$

The propagator then becomes

$$\Delta(k, -k) = \frac{1}{k \cdot k} = \frac{1}{2s_{kk}}. \quad (11)$$

The tree level 3-point function receives contributions only from the cubic vertex of (1) and, on-shell, it is simply $V_3 = \kappa c_{13}\bar{a}_{13}$. In [17], Parkes shows that this can be obtained from the usual on-shell Plebański vertex, which is $a_{13}\bar{a}_{13}$, by an $SO(2, 2)$ transformation. In our approach, since the \bar{y} and \bar{z} coordinates are “color” variables, Lorentz transformations are related to Lie algebra redefinitions of the field ω , for

⁵The quantity \tilde{c}_{ij}^2 of [4] is given by $a_{ij}\bar{a}_{ij}$. When k_i, k_j and $(k_i + k_j)$ are on-shell this becomes c_{ij}^2 .

example through commutators with q and p . For example, if we were using exactly the WZW model, the classical background would be a product of one anti-holomorphic term on the left and one holomorphic term on the right. We could choose to define the quantum fluctuations either on the left or right, or even in between these two terms. The resulting four-dimensional field theory would have looked different, and the different quantum fields would be related by Lie algebra operations which would be connected with Lorentz transformations. Of course, these quantum theories would be essentially equivalent.

The $N = 2$ string tree level 3-point amplitude is given by $g_{str} c_{13}^2$. When we compare it with our field theory amplitude, while keeping in mind the issue of Lorentz transformations, we conclude that $\kappa = g_{str}$. In [17], it is also verified that the tree level on shell 4 and 5-point functions coming from this cubic vertex vanish. Therefore, when checking these amplitudes for our model (1), we need not consider graphs where the cubic term enters alone.

We will now show directly that the tree level on shell 4-point function vanishes. Later, we will relate our model with self-dual Yang-Mills and show why the 5 and higher on-shell tree level amplitudes also vanish. The only graph we need to examine is the one with a single 4-point vertex, since the contribution of the cubic term gives zero. The term where the legs 1 and 2 are contracted, denoted by (12,34), will be given by

$$\bar{a}_{12}\bar{a}_{34}[k_{14} - k_{13} + k_{23} - k_{24}] \quad (12)$$

If we put this together with (34,12) we get

$$\bar{a}_{12}\bar{a}_{34}[s_{14} - s_{13} + s_{23} - s_{24}] = 2\bar{a}_{12}\bar{a}_{34}(s_{23} - s_{13}), \quad (13)$$

where we have used momentum conservation and the on-shell property $s_{11} = 0$, etc. The final result is obtained by summing the remaining four terms (13,24),(24,13),(14,23) and (23,14), which after using momentum conservation and the on-shell condition,

gives

$$V_4 \sim \kappa^2 [\bar{a}_{12}\bar{a}_{13}s_{23} + \bar{a}_{13}\bar{a}_{23}s_{12} - \bar{a}_{12}\bar{a}_{23}s_{13}]. \quad (14)$$

But, as a consequence of the highly constrained kinematics in $(2, 2)$ signature, the term in brackets in (14) vanishes when all momenta are on-shell [4, 17]. Therefore, the total on-shell tree level 4-point amplitude for (1) vanishes.

One could proceed and check that the 5 and higher point functions vanish on-shell. But let us show that the double scaling limit $(\Lambda \rightarrow 0, K(1 + \epsilon) \rightarrow 0)$ of the two-dimensional WZW model that we are considering, is related in a specific way to a large N limit of four-dimensional self-dual Yang-Mills theory. The vanishing of the higher point amplitudes will then be seen as a consequence of this relation.

Consider the self-dual Yang-Mills equations

$$F_{\bar{\mu}\bar{\nu}} = F_{\mu\nu} = 0, \quad \eta^{\mu\bar{\nu}} F_{\mu\bar{\nu}} = 0, \quad (15)$$

in the gauge where $A_{\bar{\mu}} = 0$. The first equation in (15) is solved by $A_{\mu} = g^{-1}\partial_{\mu}g$ so that the equation of motion, Yang's equation, reads

$$\eta^{\mu\bar{\nu}} \partial_{\bar{\nu}}(g^{-1}\partial_{\mu}g) = 0. \quad (16)$$

We now consider the Donaldson-Nair-Schiff action [21, 22]

$$S = \frac{i}{4\pi} \int d^4x \text{Tr}(g^{-1}\partial^{\mu}g g^{-1}\partial_{\bar{\mu}}g) + \frac{i}{12\pi} \int_{M_5} \omega \wedge \text{Tr}(g^{-1}dg)^3, \quad (17)$$

where Tr denotes the trace on the Lie algebra of the gauge group, $G(N)$. Parametrizing $g(x^{\mu}, x^{\bar{\mu}}) = \exp(\hat{\phi})$, one expands the action in powers of the field $\hat{\phi}$. In the limit $N \rightarrow \infty$, we have a six-dimensional non-linear scalar field theory where the matrix field $\hat{\phi}(x^{\mu}, x^{\bar{\mu}})$ becomes a scalar field $\phi(x^{\mu}, x^{\bar{\mu}}, q, p)$, where q, p are the large N color variables [14]. If we reduce by identifying $x^{\bar{1}}$ with q and $x^{\bar{2}}$ with p , that is if we impose

$$(\partial_{\bar{1}} - \partial_q)\phi = (\partial_{\bar{2}} - \partial_p)\phi = 0, \quad (18)$$

we obtain a four-dimensional theory. We mention that in the Leznov-Parkes gauge this was seen to lead to the second heavenly equation of self-dual gravity [19]. In our case, we have a non-linear theory since the vertices follow from the four-dimensional WZW action evaluated in the large N limit. Expanding (17), we have after some algebra

$$S = \int d^4x \left\{ \frac{1}{2} \phi \partial^\mu \partial_{\bar{\mu}} \phi + \frac{1}{3!} \eta^{\mu\bar{\mu}} \varepsilon^{\bar{\rho}\bar{\nu}} \phi \partial_\mu \partial_{\bar{\rho}} \phi \partial_{\bar{\mu}} \partial_{\bar{\nu}} \phi + \frac{1}{4!} \eta^{\mu\bar{\mu}} \varepsilon^{\bar{\lambda}\bar{\nu}} \varepsilon^{\bar{\rho}\bar{\sigma}} \partial_\mu \partial_{\bar{\lambda}} \phi \partial_{\bar{\nu}} \phi \partial_{\bar{\rho}} \phi \partial_{\bar{\sigma}} \phi \right. \\ \left. + \frac{1}{5!} \varepsilon^{\bar{\rho}\bar{\sigma}} \varepsilon^{\bar{\lambda}\bar{\nu}} \varepsilon^{\bar{\xi}\bar{\eta}} \partial_\mu \partial_{\bar{\rho}} \phi \partial_{\bar{\sigma}} \phi \partial_{\bar{\xi}} (\partial_\mu \partial_{\bar{\lambda}} \phi \partial_{\bar{\nu}} \phi) \partial_{\bar{\eta}} \phi + \dots \right\} \quad (19)$$

These terms agree, to this order, with the vertices of the double scaling limit of our two-dimensional model (1). This correspondence can now be used to conclude about the tree level on-shell amplitudes of our model from those of self-dual Yang-Mills theory. The latter represents (at finite N) a field theory of open $N = 2$ strings with Chan-Paton factors for the gauge group, say $U(N)$ [6]. These amplitudes are given in a factorized sum over non-cyclic permutations

$$\sum_{\sigma} \text{Tr}(T_{\sigma_1} T_{\sigma_2} \dots T_{\sigma_n}) S_n(k_1, \dots, k_n).$$

In momentum space, the reduction (18) is performed by identifying the conjugate momenta $k_q = k_{\bar{1}}$ and $k_p = k_{\bar{2}}$. At tree level, by momentum conservation at the vertices, if these relations are imposed on the external momenta they will be preserved throughout the Feynman graphs. Then, the vanishing of the open $N = 2$ string amplitudes for $n \geq 4$, implies the vanishing of the corresponding S_n 's and also of the amplitudes for the model (1), indicating that it is indeed an appropriate field theory of the closed $N = 2$ string, at least at tree level. At loop level however, there are integrations of the momenta along the loops, and this argument does not apply. We therefore proceed with a direct calculation.

3.2 The One-Loop 3-Point Function

In the usual cubic action for the field theory of the closed $N = 2$ string, the one-loop 3-point amplitude [5] is less infra-red divergent than the corresponding string amplitude, while it is also ultra-violet finite. In our case, with the action in (1), we will obtain similar results. However, as we mentioned before, the underlying $2 + 2$ structure, with two dimensions coming from color, may if further explored solve this problem.

At one-loop, the 3-point amplitude may receive contributions from several types of graphs. This is to be contrasted with the cubic theory where only one type of graph enters. Although, as we will see below, only this graph contributes also in our case, it is tempting to conjecture that the higher point vertices in (1) will be important to ensure good properties of the theory at more than one loop level. Indeed, that is the case for the usual two-dimensional WZW model. Let us examine these different diagrams each at a time.

The diagram with one 3-point tadpole vanishes because the cubic vertex is zero when two of the momenta at the vertex are equal. To examine the remaining graphs it is convenient to introduce Schwinger parameters. We will use them to find the symmetry properties of the various terms, and to find which terms will vanish. However, we note that the fact that we are in signature $(2, 2)$ makes the definition of these integrals a subtle one.

The remaining Feynman diagrams contain potentially ultra-violet divergent terms with a naive behaviour $\sim M^6, \dots, \sim M^2$, for an UV cut-off M , but as we will see these terms are in fact zero. We have the diagram with one 5-point vertex and one propagator. After considering all possible leg permutations entering the vertex,

we obtain, with ϵ a regulator for the α integral,

$$\sum_{\sigma \in S_3} \int_0^\infty d\alpha \int d^4p \exp(-\alpha\epsilon) \exp(i\alpha p^2) [-2\bar{a}_{\sigma_1 p} + \bar{a}_{\sigma_3 p}] \bar{a}_{\sigma_1 p} \bar{a}_{\sigma_2 \sigma_3} c_{\sigma_2 \sigma_3} = 0. \quad (20)$$

This vanishes because \bar{a}_{ip} contains only the $p_{\bar{y}}$ and $p_{\bar{z}}$ components of p such that when we perform the gaussian integral over d^4p we obtain zero. Similar arguments show that the graphs with one 3-point and one 4-point vertex also vanish. In addition, the ultra-violet divergent terms in the remaining diagram, the one with three 3-point vertices, also vanish for the same reasons. This is analogous to what already happens in the pure cubic theory [5].

The only surviving term is the infra-red divergent one in the last diagram above, which is proportional to

$$\kappa^3 (c_{13} \bar{a}_{13})^3 \int_0^\infty d\alpha_1 d\alpha_2 d\alpha_3 \frac{\alpha_1^2 \alpha_2^2 \alpha_3^2}{(\alpha_1 + \alpha_2 + \alpha_3)^8} \exp(-(\alpha_1 + \alpha_2 + \alpha_3)\epsilon) \left(\int d^4p \exp(ip^2) \right). \quad (21)$$

We stress that one should be careful in interpreting the integrals in (21), since it is not clear which regularizing prescription to use, because of the peculiarities of the $(2, 2)$ signature.

The Schwinger parameter integration in (21) gives an infra-red divergence of the form

$$\int_\epsilon ds \frac{1}{s^3} \sim \frac{1}{\epsilon^2}$$

where ϵ is some infra-red cut-off. This is indeed equivalent to the result of [5]. However, one also has to note an infra-red divergence associated with the singularity due to the $(2, 2)$ metric. In fact, the gaussian momentum integral in (21) is also divergent. In the present approach, two of the momentum components come from large N color, $(k_q, k_p) = (2\pi n_q/N, 2\pi n_p/N)$. This defines a natural infra-red regulator $\epsilon = 2\pi/N$. The transition from the sum over n_q, n_p to the integral $\int dq dp$ then involves a factor of N^2 which is equivalent to $1/\epsilon^2$. The total dependence on the infra-red regulator would then be $1/\epsilon^2 \cdot 1/\epsilon^2 = 1/\epsilon^4$ which would agree with the $N = 2$ string one.

4 Conclusions

In this paper, we start the exploration of the quantum properties of four-dimensional self-dual gravity viewed as a large N two-dimensional WZW sigma model. We have shown how the expansion around a non-trivial classical background for the two-dimensional sigma model yields a four-dimensional field theory living in a space-time of signature $(2, 2)$. In this approach, we generate an infinite sequence of higher point vertices, which can be determined systematically from the two-dimensional sigma model. At tree level, we have seen that our model is related to a particular dimensional reduction of four-dimensional large N self-dual Yang-Mills theory. As such, it is closely related to a conjecture of Ooguri and Vafa (last paper of [4]).

We have checked that the amplitudes of our model are consistent with the amplitudes of the closed $N = 2$ string, after we allow for a Lorentz $SO(2, 2)$ transformation. The 3-point amplitude at one-loop level in our model, is similar to the one of the usual cubic actions for self-dual gravity, with ultra-violet finiteness and an infra-red divergence that is weaker than the one from the string. We believe that our approach may contain the solution to this puzzle, through a careful examination of the large N limit or maybe through a more specific choice of Poisson algebra taking into account the global topology of the surface Σ . Although this naive argument should of course be further substantiated, it provides an example of how a better agreement between the field theory and the string could be achieved at the quantum level in this framework. Most importantly, given the good quantum properties of the WZW model, we expect that the infinite series of terms in our lagrangian is likely to have important and desirable consequences at higher loop level.

It would be interesting to apply this large N approach in the case where worldsheet instantons (from the Maxwell field in the $N = 2$ worldsheet supergravity multiplet) are included in the string amplitudes [4]–[9] and achieve a better understanding

of the four-dimensional Lorentzian properties of these field theories.

We note that a similar approach should also work in the case of the open string. Indeed, classical self-dual Yang-Mills theory can also be obtained from a large N two-dimensional sigma model, where one can generate Chan-Paton factors or gauge groups in four dimensions if one extends the Poisson algebra $sdiff(\Sigma)$ by the Lie algebra of the chosen gauge group [16]. Related considerations would allow the extension to the heterotic string as well [4, 11].

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